Latent Variable Hybrids: Overview Of Old And New Models

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Overview

	Continuous Latent Variables	Categorical Latent Variables	**Hybrids**
Cross-Sectional Models	Factor analysis, SEM	Regression mixture analysis, Latent class analysis	**Factor mixture analysis**
Longitudinal Models	Growth analysis (random effects)	Latent transition analysis, Latent class growth analysis	**Growth mixture analysis**

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Overview Of Hybrids: Modeling With Categorical And Continuous Latent Variables

Measurement Invariance

Yes — Factor Analysis Emphasis

No — Cluster Analysis and Factor Analysis Emphasis

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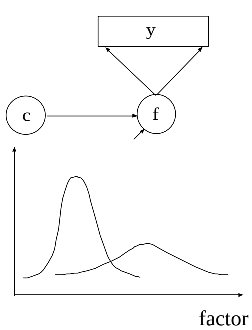
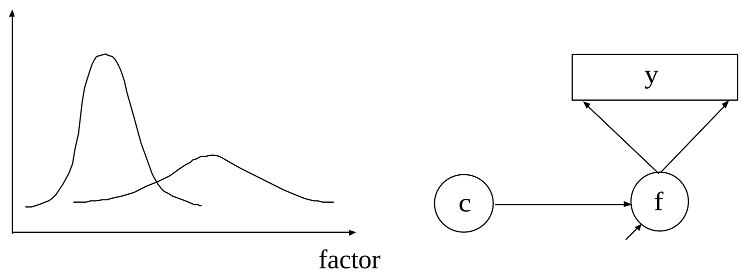
Parametric Factor Distribution

Yes — Mixture Factor Analysis

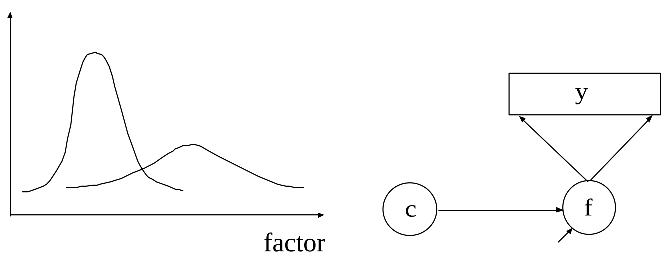


Overview Of Hybrids: Measurement Invariance, Parametric Factor Distribution



- Cross-sectional examples:
  - Mixture factor analysis (McDonald, 1967, 2003; Muthen, 1989; Yung, 1997; Lubke & Muthen, 2005)

Overview Of Hybrids: Measurement Invariance, Parametric Factor Distribution



- Longitudinal examples:
  - Growth mixture modeling of trajectory classes (Verbeke & LeSaffre, 1996; Muthen & Shedden, 1999)
  - Intervention effects varying across trajectory classes (Muthen et al, 2002)
  - Regime (latent class) switching (Dolan, Schmittman, Lubke, Neale, 2005)

Overview Of Hybrids: Modeling With Categorical And Continuous Latent Variables

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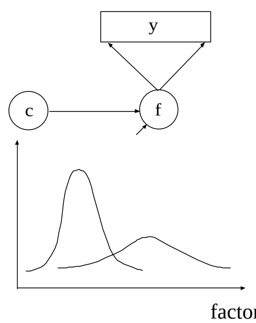
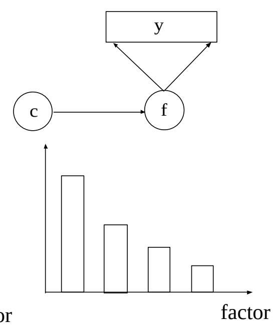
Parametric Factor Distribution

Yes — Mixture Factor Analysis

No — **Non-Parametric FA**





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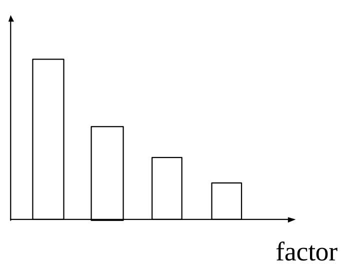
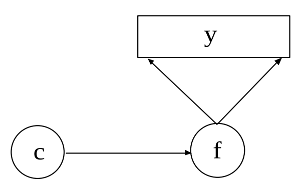
Overview Of Hybrids: Non-Parametric Factor Distribution





- Cross-sectional examples:
  - Latent class factor analysis for DSM-V (Muthen & Asparouhov, 2006)
  - IRT (?)

- Longitudinal examples:
  - Binary growth; non-normal random effects (Aitkin, 1999)
  - Trajectory groups for criminal offenders (Nagin & Land, 1993)

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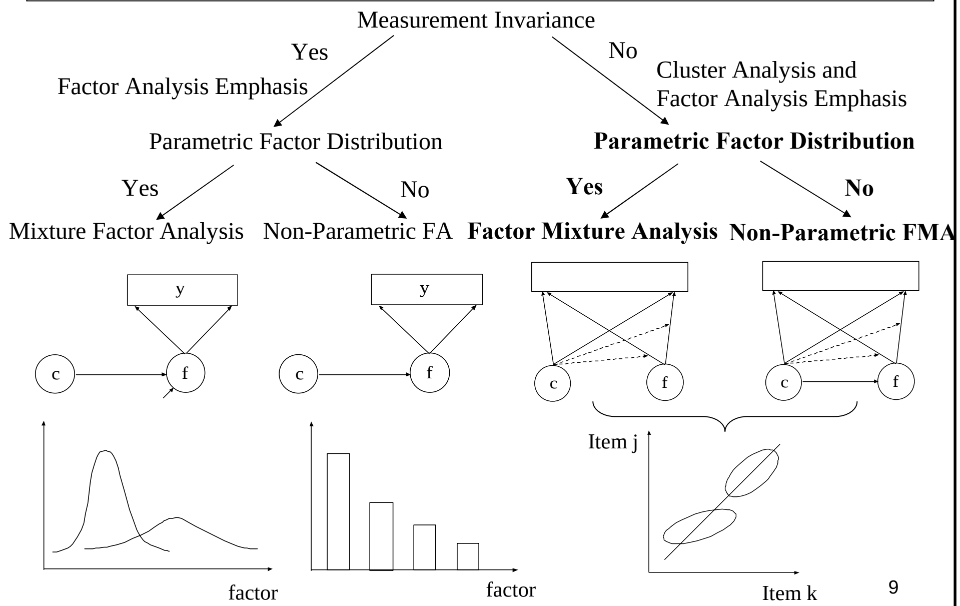
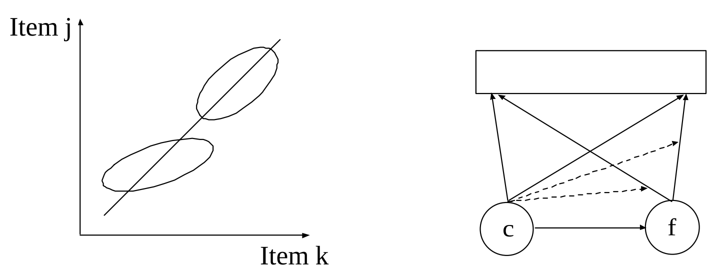
Overview Of Hybrids: Modeling With Categorical And Continuous Latent Variables

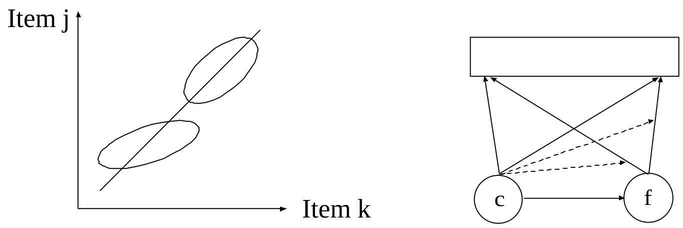


Overview Of Hybrids: Factor Mixture Analysis



- Cross-sectional examples:
  - Factor mixture analysis in psychometrics (Blafield, 1980; Yung, 1997)
  - Structural equation mixtures; market segmentation (Jedidi, Jagpal & DeSarbo, 1997)
  - IRT mixtures; solution strategies, guessing, levels of difficulty - Saltus (Yamamoto, 1987; Mislevy & Verhelst, 1990; Mislevy & Wilson, 1996; Wilson, de Boeck, Acton, 2005)

Overview Of Hybrids: Factor Mixture Analysis (Continued)



- Cross-sectional examples (continued):
  - Factor mixture analyzers; continuous micro-array expression data (McLachlan, Do & Ambroise, 2004)
  - Factor mixture modeling; binary diagnostic criteria; genetics for twins, siblings (Muthen, Asparouhov, Rebollo, 2006)
  - Classic normal finite mixtures; Fisher's Iris data
  - Non-parametric factor mixture analysis (?)
- Longitudinal examples:
  - Factor mixture latent transition analysis (Muthen, 2006)

Unclassified Contributors

- Dayton, Meehl, Vermunt
- Suggestions welcome

Model Testing Issues

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NLSY 1989: Latent Class Analysis Of DSM-III-R Alcohol Dependence Criteria (n = 8313)

Source: Muthén & Muthén (1995)

	Latent Classes				
	Two-class solution[1]		**Three-class solution[2]**		
	I	II	I	II	III
Prevalence	0.78	0.22	0.75	0.21	0.03
DSM-III-R Criterion	Conditional Probability of Fulfilling a Criterion				
Withdrawal	0.00	0.14	0.00	0.07	0.49
Tolerance	0.01	0.45	0.01	0.35	0.81
Larger	0.15	0.96	0.12	0.94	0.99
Cut down	0.00	0.14	0.01	0.05	0.60
Time spent	0.00	0.19	0.00	0.09	0.65
Major role-Hazard	0.03	0.83	0.02	0.73	0.96
Give up	0.00	0.10	0.00	0.03	0.43
Relief	0.00	0.08	0.00	0.02	0.40
Continue	0.00	0.24	0.02	0.11	0.83

[1]Likelihood ratio chi-square fit = 1779 with 492 degrees of freedom
[2]Likelihood ratio chi-square fit = 448 with 482 degrees of freedom

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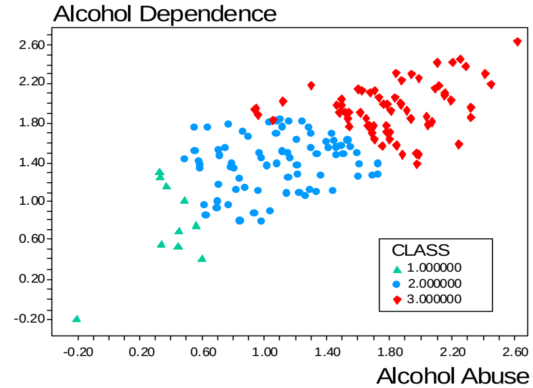
Estimated Factor Scores From Two-Factor Model By Class



LCA, 3 classes: logL = -14,139, 29 parameters, BIC = 28,539

FA, 2 factors: logL = -14,083, 26 parameters, BIC = 28,401

FMA 2 classes, 1 factor, loadings invariant:

logL = -14,054, 29 parameters, BIC = 28,370

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Deciding On The Number Of Classes: Bootstrapped LRT

- Nylund, Muthen and Asparouhov (2006) simulation study
- BLRT has better Type I error than NCS and LMR
- BLRT finds the right number of classes better than BIC and LMR

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Nylund, Asparouhov And Muthen (2006)

Latent class analysis with categorical outcomes

Model	n	BIC Classes 3	4	5	LMR Classes 3	4	5	BLRT Classes 3	4	5
10-Item	200	92	**8**	0	34	**43**	9	16	**78**	6
(Complex	500	24	**76**	0	9	**72**	14	0	**94**	6
Structure)	1000	0	**100**	0	2	**80**	17	0	**94**	6

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Factor (IRT) Mixture Example: The Latent Structure Of ADHD

- UCLA clinical sample of 425 males ages 5-18, all with ADHD diagnosis
- Subjects assessed by clinicians:
  1) direct interview with child (> 7 years),
  2) interview with mother about child
- KSADS: Nine inattentiveness items, nine hyperactivity items; dichotomously scored
- Families with at least 2 ADHD affected children
- Parent data, candidate gene data on sib pairs

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The Latent Structure Of ADHD (Continued)

Inattentiveness Items:	Hyperactivity Items:
‘Difficulty sustaining attn on tasks/play’	‘Difficulty remaining seated’
‘Easily distracted’	‘Fidgets’
‘Makes a lot of careless mistakes’	‘Runs or climbs excessively’
‘Doesn’t listen’	‘Difficulty playing quietly’
‘Difficulty following instructions’	‘Blurts out answers’
‘Difficulty organizing tasks’	‘Difficulty waiting turn’
‘Dislikes/avoids tasks’	‘Interrupts or intrudes’
‘Loses things’	‘Talks excessively’
‘Forgetful in daily activities’	‘Driven by motor’

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The Latent Structure Of ADHD: Model Fit Results

Model	Likelihood	# Parameters	BIC	BLRT p value for k-1 classes
LCA – 2c	-3650	37	7523	0.
LCA – 3c	**-3545**	**56**	**7430**	**0.**
LCA – 4c	-3499	75	7452	0.
LCA – 5c	-3464	94	7496	0.
LCA – 6c	**-3431**	**113**	**7547**	**0.**
LCA – 7c	-3413	132	7625	0.27

LCA-3c is best by BIC and LCA-6c is best by BLRT

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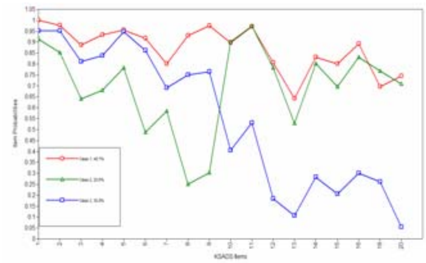
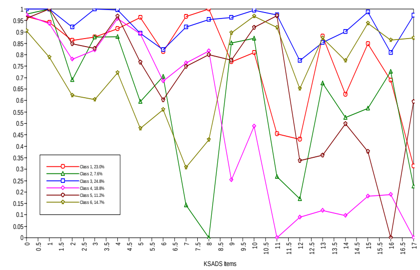
Three-Class And Six-Class LCA Item Profiles





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LCA – 7c	-3413	132	7625	0.27
EFA – 2f	**-3505**	**53**	**7331**	

The EFA model is better than LCA - 3c, but no classification of individuals is obtained

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The Latent Structure Of ADHD: Model Fit Results

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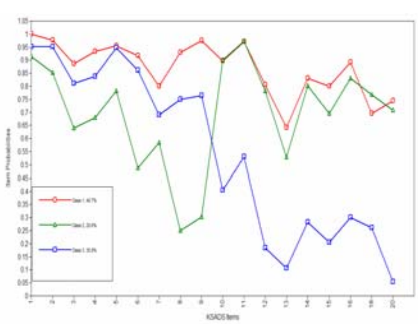
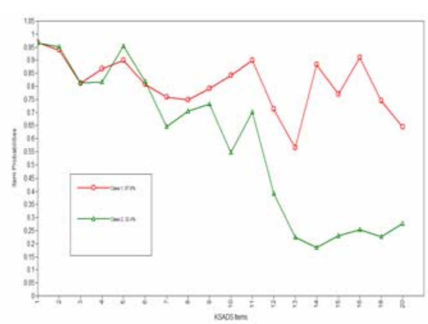
EFA – 2f	**-3505**	**53**	**7331**	
FMA – 2c, 2f	-3461	59	7280	
FMA – 2c, 2f Class-varying Factor loadings	**-3432**	**75**	**7318**	**χ^2-diff (16) = 58 p < 0.01**

Three-Class LCA And Two-Class, Two-Factor FMA Item Profiles





Factor Mixture Modeling

Categorical outcomes plus continuous-normal latent variables have the statistical and computational disadvantage of

- normality assumption
- heavy computations due to numerical integration

Non-parametric latent variable distribution avoids the normality assumption and at the same time the computational disadvantage!

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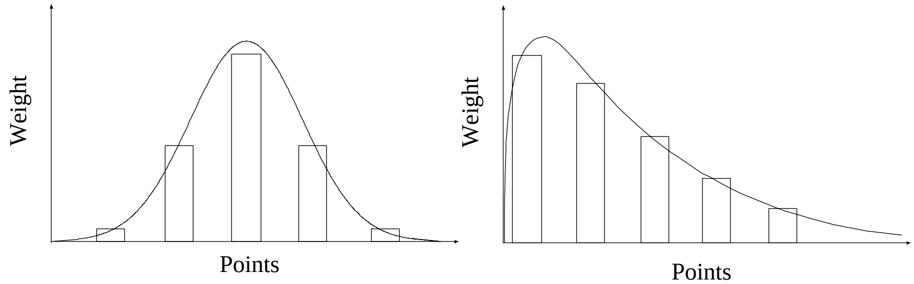
Non-Parametric Estimation Of The Random Effect Distribution Using Mixtures



Estimated weights and points
(class probabilities and class means)

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Longitudinal Analysis

- Factor analysis generalizes to random effects repeated measures (growth) analysis
- Latent class analysis generalizes to latent transition analysis
- Factor mixture analysis generalizes to growth mixture modeling and generalized latent transition analysis

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Growth Mixture Modeling: Shapes of Growth Curves

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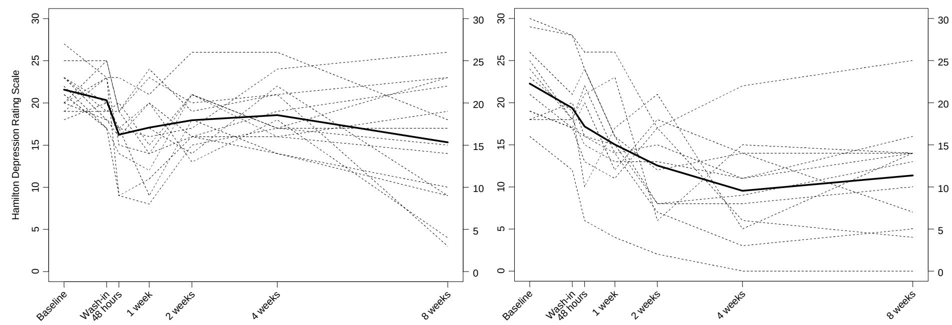
A Clinical Trial Of Depression Medication: Two-Class Growth Mixture Modeling

Placebo Non-Responders, 55% | Placebo Responders, 45%



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Example: Mixed-Effects Regression Models For Studying The Natural History Of Prostate Disease

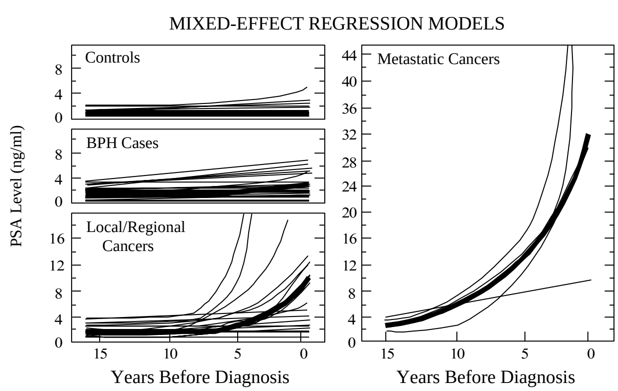

Figure 2. Longitudinal PSA curves estimated from the linear mixed-effects model for the group average (thick solid line) and for each individual in the study (thin solid lines)

Source: Pearson, Morrell, Landis and Carter (1994), <u>Statistics in Medicine</u>

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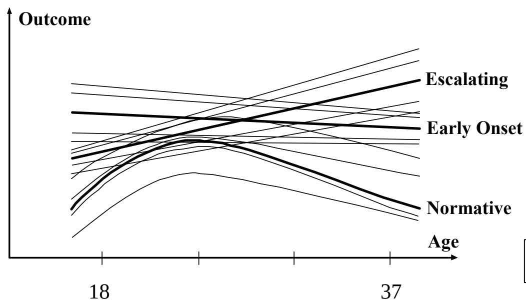
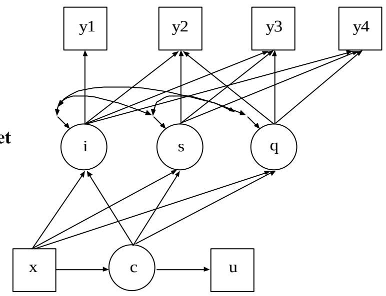
Growth Mixture Modeling
Of Developmental Pathways





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Multilevel Growth Mixture Modeling

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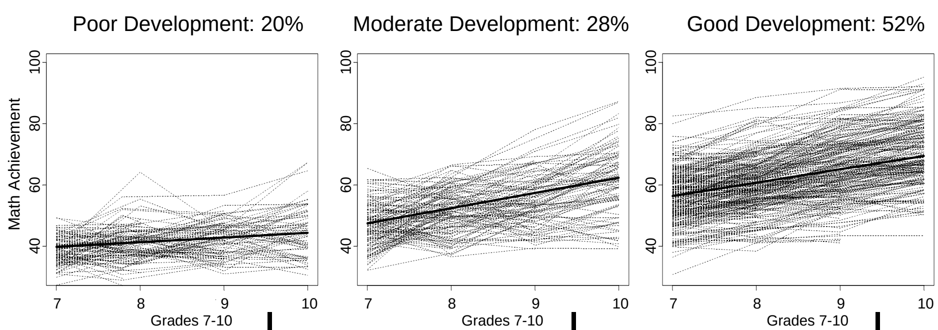
Growth Mixture Modeling: LSAY Math Achievement Trajectory Classes And The Prediction Of High School Dropout



Dropout:	69%	8%	1%

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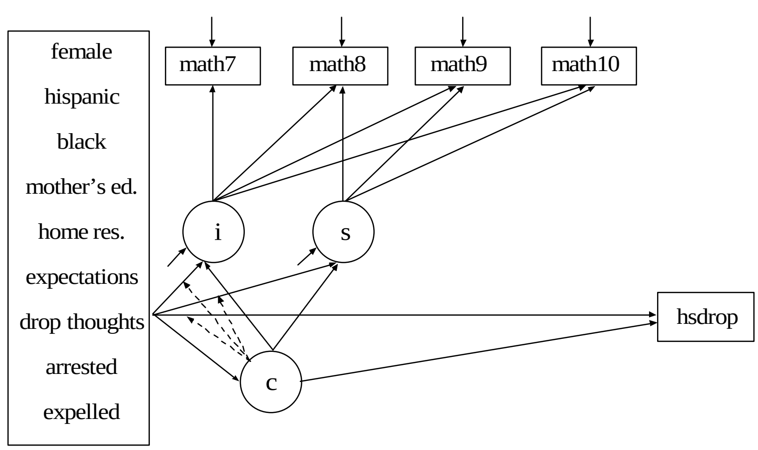

Muthen (2004)

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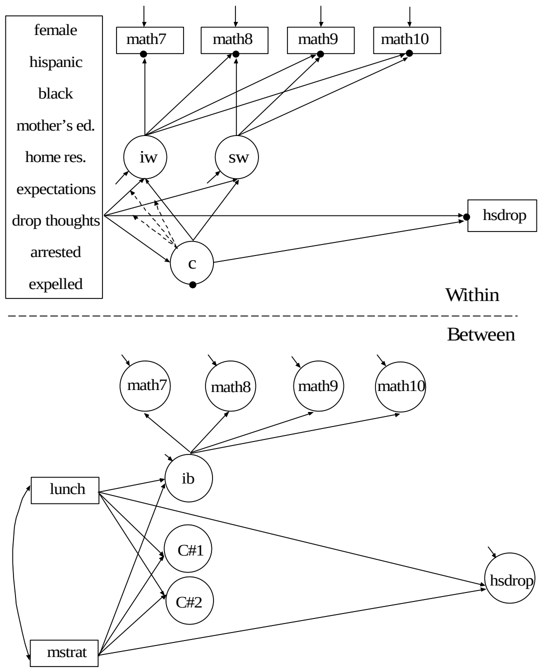

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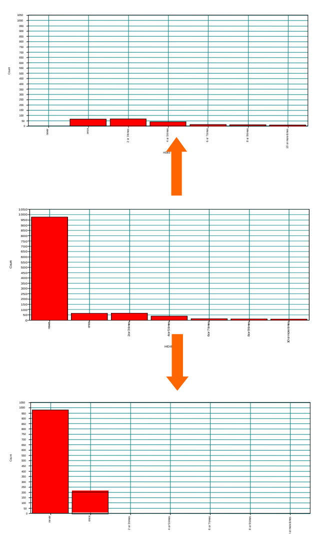
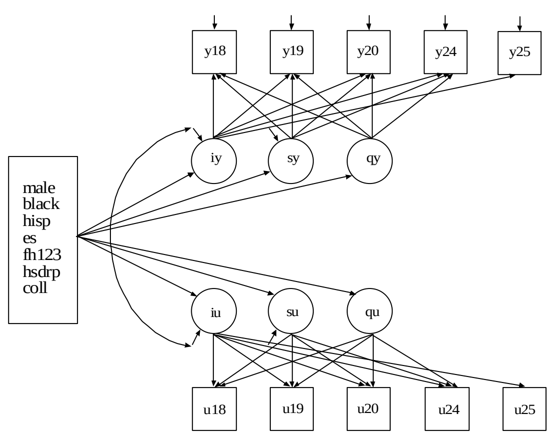
Two-Part Growth Modeling Of Frequency Of Heavy Drinking Ages 18 – 25





Olsen and Schafer (2001)

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Two-Part Modeling Extensions In Mplus

- Growth modeling
  - Distal outcome
  - Parallel processes
  - Trajectory classes (mixtures)
  - Multilevel
- Factor analysis
  - Mixtures
    - Latent classes for binary and continuous parts may be incorrectly picked up as additional factors in conventional analysis
  - Multilevel

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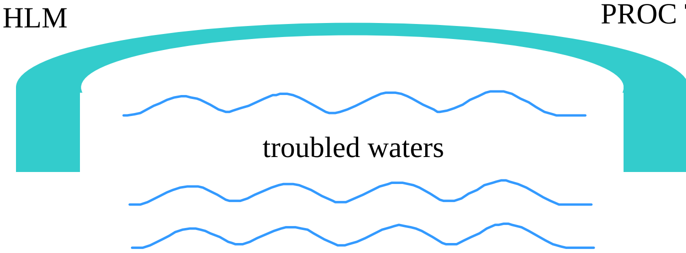
Growth Modeling Paradigms: Debate In Criminology And Annals AAPSS

Growth Mixtures

Random Effects	Muthen	Latent Classes
Raudenbush	Mplus	Nagin
HLM		PROC Traj



Sampson & Laub

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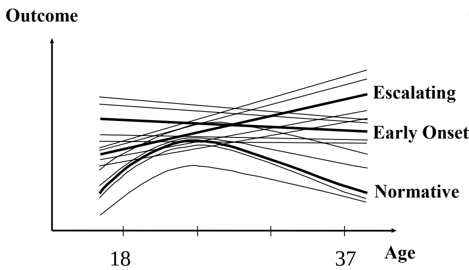
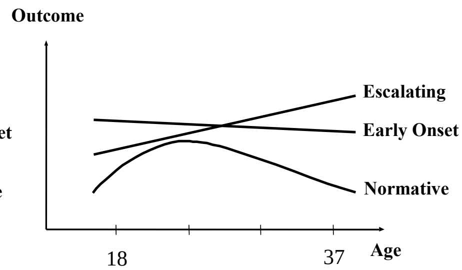
Growth Mixture Modeling Versus Latent Class Growth Analysis

GMM



Muthen

LCGA



Nagin

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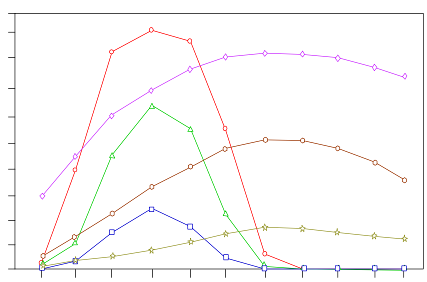
Substantive And Non-Parametric Latent Classes

- Nagin's inconsistency: Latent classes used as
  - Substantively distinct and meaningful subgroups
  - Non-parametric representation of the latent variable distribution
- Resolution: Combine substantive and non-parametric latent classes
  - Non-parametrically described variation within substantive themes
  - Easy to set up in Mplus using two latent class variables



3 non-parametric classes within each of 2 substantive classes

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Latent Transition Analysis

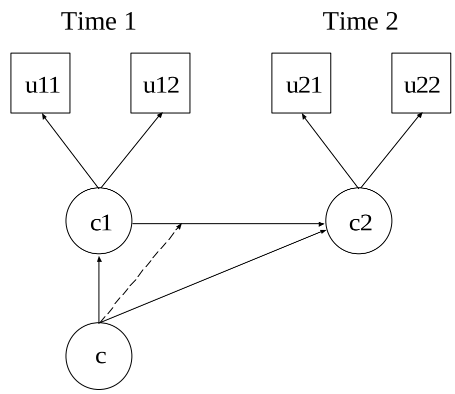
Transition Probabilities

Mover Class (c=1)

		c2 = 1	c2 = 2
c1	1	0.6	0.4
	2	0.3	0.7

Stayer Class (c=2)

		c2 = 1	c2 = 2
c1	1	0.90	0.10
	2	0.05	0.95



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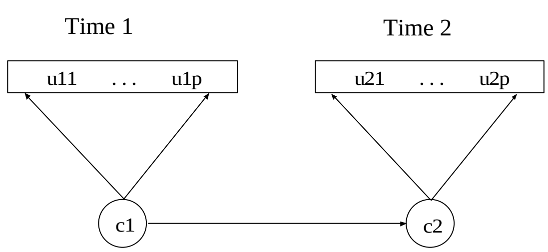
Latent Transition Analysis



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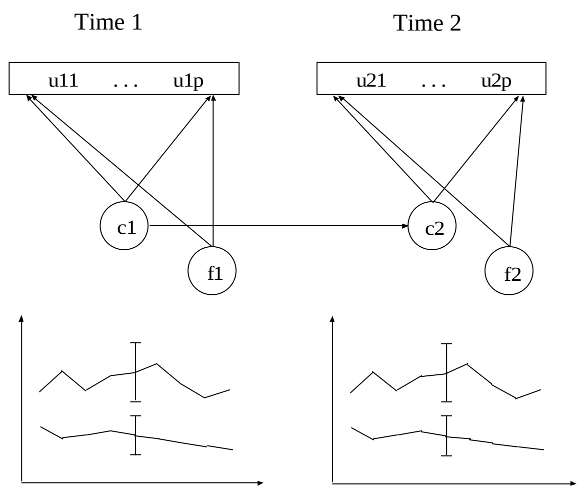
Factor Mixture Latent Transition Analysis



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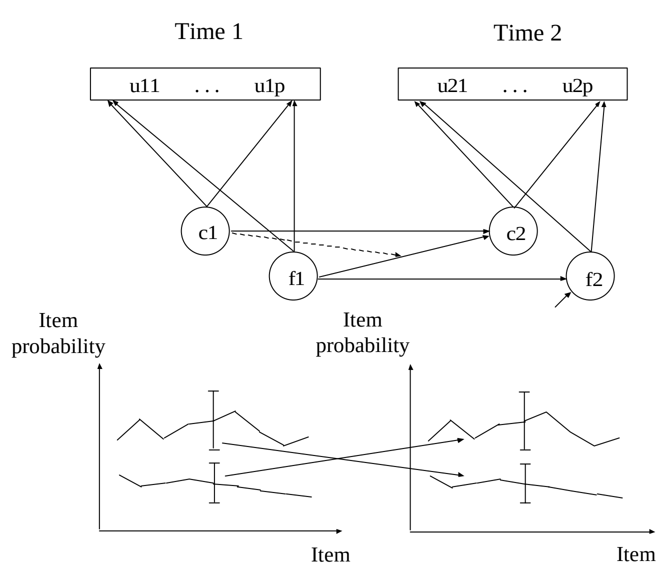
Factor Mixture Latent Transition Analysis



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Factor Mixture Latent Transition Analysis: Aggressive-Disruptive Behavior In The Classroom

- 1,137 first-grade students in Baltimore public schools
- 9 items: Stubborn, Break rules, Break things, Yells at others, Takes others property, Fights, Lies, Teases classmates, Talks back to adults
- Skewed, 6-category items; dichotomized (almost never vs other)
- Two time points: Fall and Spring of Grade 1
- For each time point, a 2-class, 1-factor FMA was found best fitting

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Factor Mixture Latent Transition Analysis: Aggressive-Disruptive Behavior In The Classroom (Continued)

Model	Loglikelihood	# parameters	BIC
Conventional LTA	-8,649	21	17,445
FMA LTA factors related across time	-8,012	40	16,306

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Factor Mixture Latent Transition Analysis: Aggressive-Disruptive Behavior In The Classroom (Continued)

Estimated Latent Transition Probabilities, Fall to Spring

Conventional LTA		
	Low	High
Low	0.93	0.07
High	0.17	0.83
FMA-LTA		
	Low	High
Low	0.94	0.06
High	**0.41**	**0.59**

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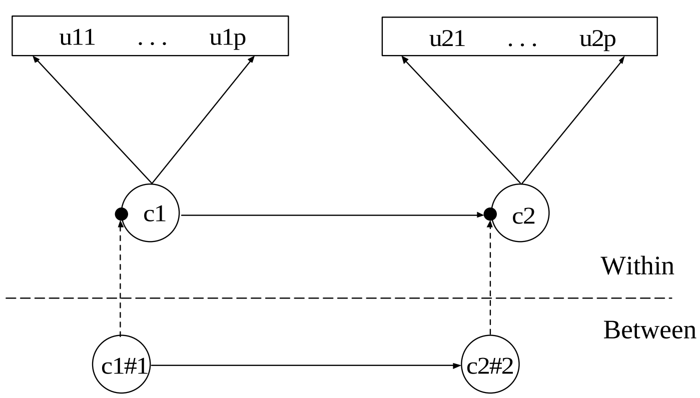
Two-Level Latent Transition Analysis



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Mplus Website

Visit www.statmodel.com to check out

- Web videos of courses

- Recent papers

- Version 4 User's Guide

- Mplus Discussion

- Short course announcements

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